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The application of graphical representations in estimation of probabilistic events

Mirian Agus
University of Cagliari

Maria Pietronilla Penna
University of Cagliari

Maribel Peró-Cebollero
University of Barcelona

Joan Guàrdia-Olmos
University of Barcelona

Eliano Pessa
University of Pavia

Abstract
The communication of uncertain and certain information is essential for individual behaviour and decision making. But many subjects have difficulty in understanding uncertainty-related problems. This difficulty can depend on the way in which materials are presented and communicated: for example, whether this is verbal-numerical or graphical. We measured subjects' performances when solving uncertainty-related problems in relation to the format of problem presentation, aiming to detect the graphical facilitation effect potentially produced by Euler-Venn and Iconic diagrams. Through a quasi-experimental mixed design, we analysed the performance of 229 undergraduate students in solving two paired uncertainty-related problems under conditions of time pressure. Applying a non-parametric statistical data analysis, we found a graphical facilitation effect, particularly with Euler-Venn diagrams.

Keywords: reasoning on uncertainty, imagery, graphical representation, time pressure, risk communication

Introduction

In daily social relations, much information is communicated with varying amounts of certainty and uncertainty. These dimensions play a significant part in the understanding of the social world and in the organization of decision making in the usual circumstances. It is well known that certainty and uncertainty are coded in each statement, in particular when this statement is related to reliability, probability, plausibility and likelihood.

In recent decades we have observed the rising of a strong interest in probabilistic reasoning in problem-solving and in judgement making in uncertain situations. Indeed, there is a considerable body of research on the evaluation of biases in the assessment of risks and probabilities of events in everyday contexts. When a subject makes interpretations concerning dubious, vague or indefinite situations, under conditions of limited time, he will typically be unable to make the most correct estimation, even if the solution may be reached through reasoning (Ibrekk & Morgan, 1987).

Reasoning on uncertainty is defined as a specific type of statistical reasoning. In general, the concept of statistical reasoning is described as the 'way people reason with statistical ideas and make sense of statistical information' (Garfield & Chance, 2000, p.101). Through statistical reasoning, subjects have the capability to implement and create solutions for problems, both day-to-day and academic.

On the basis of these assertions, Garfield (2003) supposes that six kinds of well-defined reasoning can be recognized in statistical reasoning: on data, on representations of data, on uncertainty, on association, on statistical measures and on samples. Each type of reasoning could be elicited by specific activities related to data comprehension, analysis and communication. In particular, reasoning on uncertainty is defined as 'understanding and using ideas of randomness, chance, likelihood to make judgments about uncertain events; knowing that not all outcomes are equally likely' (Garfield, 2003, p. 25; Garfield & Chance, 2000).

Reasoning on uncertainty is an important tool to be applied when we have to assess variability, find relationships between events and making appropriate decisions regarding incertitude (e.g. Batanero & Godino, 2005).

Many studies have shown that individuals make mistakes when they reason about probabilities. This circumstance is related to the fact that they are not familiar with the probabilistic information and therefore have difficulty in understanding it (Gigerenzer & Goldestein, 1996; Van Dijk & Zeelenberg, 2003). When subjects try to use uncertain information, they have a tendency to incorrectly estimate the probability of events. It is relevant to highlight that this occurs in both expert and non-expert subjects (Camerer et al., 1997).

Previous research has shown that probabilistic statements can be supposed to be related to problem organization and representation. Investigations on this topic affirm that problem representation brings up to the nature of thinking that subjects need to use to modelling and solving a specific problem (Bennet et al., 2000). Daniel and Embretson (2010) introduced a model in order to show that the perception of problem complexity is dependent on the following specific dimensions: the type of problem presentation, the amount of information provided and the specific subject's knowledge. Many researchers feel that an effective representation of a probabilistic and uncertainty-related problem can be decisive in reaching a favourable solution. Other researchers remark that probabilistic reasoning is also related to information organization (e.g. Girotto & Gonzalez, 2001).

Several researchers (Feeney et al., 2004; Webber & Feeney, 2004) studied the effect of graphical representation on reasoning. They highlighted that people reorder, in their mental representation, the information presented in a graphical way. This reordering tendency appears to be related to individual spatial skills.

This could indeed be related to a subject's specific cognitive style, which is the way in which their cognitive system operates and how information is acquired and processed. These individual styles influence the ways in which individuals perceive, remember, think, organize and solve particular problems (Kozhevnikov et al., 2002, 2005; Paivio, 1971; Presmeg, 1986; Sternberg & Grigorenko, 1997).

Garcia-Retamero, Galesic, and Gigerenzer (2011) reported that the occurrence of differences in the understanding of probabilistic problems is related to the format in which the problem is presented: whether it is verbal-numerical or graphical-pictorial. Much previous work has shown that the use of several visual aids appears to be an important factor in clarifying uncertainty-related problems and in improving probabilistic reasoning (Tversky et al., 2000; Zacks et al., 2002).

Graphs are important for the subjects because they are powerful tools, enabling to communicate and summarize information in an effective way. Some research (e.g. Kirschenbaum & Arruda, 1994; Stone et al., 1997) highlighted that graphical displays of uncertain data are often better than verbal descriptions in favouring the correct solution of probabilistic problems. Moreover, other authors supported the utilization of pictures as a useful representational method to help solving statistical problems (e.g. Brase, 2009; Moro & Bodanza, 2010). In this regard Ibrekk and Morgan (1987) studied the effectiveness of particular kinds of graphical representation for communicating weather forecasts. The important role of picture-based representations in helping to solve probabilistic problems has been often mentioned by a number of other authors (e.g. Lipkus, 2007; Lipkus & Hollands 1999; Zimmermann & Cunningham, 1991).

Within this context it is to be remembered that Owens and Clements (1998) already showed that visual imagery plays an important role in creating the connotation of a problem and applying problem-solving methods. Besides, visual imagery influences in a strong way reasoning constructions.

In order to account for this circumstance, Wild and Pfannkuch (1999) introduced *transnumeration* as one of the essential modes of statistical reasoning, which allows one to obtain information in a data set when switching from one representation to another within a given system. In this sense, graphs are *transnumeration* instruments. To define *transnumeration*, Wild and Pfannkuch (1999, p. 227) assert: 'The most fundamental idea in a statistical approach to learning is that of forming and changing data representations of aspects of a system to arrive at a better understanding of that system. We have coined the word *transnumeration* to refer to this idea. We define it as numeracy transformations made to facilitate understanding. *Transnumeration* occurs when we find ways of obtaining data (through measurement or classification) that capture meaningful elements of the real system. It pervades all statistical data analysis, occurring every time we change our way of looking at the data in the hope that this will convey new meaning to us. We may look through many graphical representations to find several really informative ones'.

We can list many features of graphs that affect their capability to transmit information. For instance, Tufte (2001) affirmed that colour and length play an important role in the understanding of pictorial representations. Bennet and colleagues (2000) also highlighted other aspects of graphical representations that could have an important role in reasoning on representations, such as: the specific object used (e.g. line, triangle, square), the dimensional characteristics (e.g. length, width, slope, area, perimeter), the logical operators (e.g. greater than, less than) and the reported values. All these elements contribute to determining the subjective interpretation of a specific graphical representation. For this reason different graphs, and graphs structured in a different way, could have different effects on reasoning.

Moreover, these features imply that the use of images of different types could involve different visuo-spatial abilities. The type of picture used in the problem illustration significantly affects the subject's responses and, indeed, his choice and application of strategies of solution.

Ibrekk and Morgan (1987), in their work with non-technical people, found that background knowledge of statistics did not enhance performance in graphical reasoning on uncertainty. This could be related to the fact that often existing approaches to statistical training do not highlight the use of graphical methods of displaying information.

In reference to these themes, on the other hand, Knauff and Johnson-Laird (2002) affirm that visual aids could impede reasoning in some probabilistic problems, because these processes require the individuation of complex multiple relations in order to construct a mental model. The mental model must capture the relevant logical proprieties of the problem, abstracting away from visual details (as colours, textures), then avoiding to include the worthless specificities. In this perspective, then, the visual data could overload the cognitive system with additional and insignificant particularities that might oppose the inference. In fact, several researchers (Kellen et al., 2007; Knauff et al., 2003; Ruff et al., 2003; Schmitz, 2005) have highlighted that spatial processes can prevent successful reasoning when these processes are executed at the same time.

This paper represents an initial stage of our research investigating the use of imagery in statistical reasoning on uncertainty in order to detect the effect of graphical facilitation (Moro et al., 2011; Penna et al., 2012). In this stage we try to explore how the performance of this kind of reasoning is influenced by two different forms of problem presentation: verbal-numerical and graphical. In this regard we compared pairs of problems with the same structure but different presentation format in order to assess how much the strategies of solution depend on the adopted presentation format. This assessment is based on measured subjects' performances. In order to better investigate the effects on performance produced by specific features of graphical presentations we used two different kinds of graphical descriptions: the Euler-Venn and the Iconic diagrams.

More precisely, the research questions were:

a. Is reasoning on uncertainty influenced by the presence versus absence of graphical presentations?
b. Have the Euler-Venn diagrams and the Iconic diagrams the same effect on reasoning on uncertainty?

Within our research the subjects were required to give specific responses to daily problems in realistic situations. The aim was to identify the subjects' ability to precisely transfer features of the underlying information and to make the most careful decisions possible, based on the accessible evidence. The used procedure was based on a sort of simulation of real life situations in which subjects were given likelihood information using both presentation formats (verbal-numerical and graphical), and they were requested to take ensuing decisions.

Past research has rarely investigated subjects' solution skills related to such problems by proposing a design for repeated measures. In reality, most researches compared the solution ability in different subjects in relation to different problems presented in the verbal-numerical and graphical-pictorial formats. Nevertheless, we are interested in investigating the role of imagery in reasoning on uncertainty and, for this reason, it was important to offer evidence matched in the same subject and compare the solution ability in two ways. Moreover, in order to better understand the contribution of the presentation format, it was important to present simple paired problems that would not require specific statistical skills, but would necessitate only a basic procedural mathematical ability. For these reasons, we chose a type of uncertainty problem which did not refer to specific conceptual knowledge and statistical expertise (Godino et al., 2007). These problems appear suitable and useful for assessing procedural knowledge in the practical resolution of uncertainty problems. They are similar to the problems commonly used in past researches in order to study this type of reasoning (e.g. Moro et al., 2011).

The fall-out of our research, as well as of all researches on the role played by problem presentation format, is given by the individuation of an appropriate way to improve and enhance the communication of uncertainty in the daily contexts. For these reasons we assessed the accuracy of decisions in uncertainty problems related to the ordinary context. Moreover our subjects solved the problems under conditions of time pressure, as is often the case in everyday situations. Of course, this fact could intensify the estimation biases in understanding and decision making (Penna et al., 2012).

Method

Participants

We recruited 229 undergraduate Italian students (female 84.3%; age range 18–40 years, m=22.19, ds=4.3) from the Faculty of Educational Sciences of the University of Cagliari (Italy).

The problems were given in paper-and-pencil format in a large group, in a classroom. They were administered to subjects in a single step at the beginning of the second half academic semester (from October to November 2011). The subjects voluntarily participated in the research. The sampling was not probabilistic, but we applied quota selection – stratification using predetermined quotas in relation to the variables of gender and age.

Instruments and procedure

The instrument was extrapolated and adapted from bibliography (Garfield, 2003). It was administered in the Italian language.

Each subject was given two pairs of problems investigating reasoning on uncertainty related to everyday contexts, selected from Garfield's Statistical Reasoning Assessment (SRA; 2003). The SRA is a multiple-choice examination, consisting of 20 requests (Garfield, 2003; Konold, 1989). The complete questionnaire is structured into Correct Reasoning and Misconception scales (eight for each dimension). Previous studies analysed the entire correct reasoning score and the overall misconceptions score, but these measures do not seem to assess a single trait (test-retest reliability reported as 0.7 and 0.75: Tempelaar, 2004; Tempelaar et al., 2007). The two items designated from the questionnaire (matching the second and third items in the SRA) were selected because they are connected to other problems typically used in the bibliography of research in this field. For these problems we built a verbal-numerical adaptation and a corresponding version in graphical format.

The first problem concerned the understanding of disease risk related to a medical treatment, while in the second problem the subjects were requested to evaluate the accuracy of probabilistic weather-forecasting (see Figure 1).

The problems were counterbalanced and were presented in each session in randomized order. Specifically, randomly half of the sample solved before the problems in the verbal numerical (N) and later in the graphical form (G), then and there in order N-G; the remaining students solved the problems in the reverse order (G-N). In each case, half of student solved the problems in two different reversed sequences.

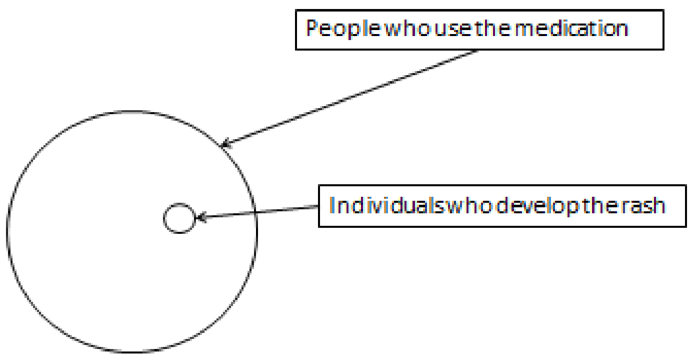
The items required the individuation of the correct response among five options.

Item 1
VERBAL-NUMERICAL FORMAT
The information below is included in the form of informed consent presented to a patient before undergoing cosmetic surgery with injections of botulin toxin.
Warning: After the implementation of the intervention, there is a 15% chance of developing an allergic reaction.
Which of the following phrases may be the best interpretation of this warning?

a. Do not have the surgery if it is not necessary, there is a good chance of developing an allergic reaction
b. Following the intervention, 15% of the surgically treated areas may experience an allergic reaction
c. If an allergic reaction develops, it will probably involve only 15% of the body
d. About 15 of 100 people who do this intervention develop an allergic reaction
e. There is a low probability of developing an allergic reaction following the implementation of the intervention.

GRAPHICAL FORMAT
The information below is included in the package leaflet of the medicinal product.
Warning: In the application of the medication on the skin, there is a 15% chance of developing a rash. If a rash appears, consult your doctor.
Which of the following phrases may be considered the best interpretation of this warning?

a. Do not use the medication on the skin, there's a good chance of developing a skin rash
b. For topical application, apply only 15% of the recommended dose
c. If a rash develops, it will probably involve only 15% of the skin
d. About 15 of 100 people who use this medication develop a skin rash
e. There is little chance of getting a rash using this medication



Item 2
VERBAL–NUMERICAL FORMAT
A meteorological center wanted to determine the accuracy of its forecasts. For this, it searched documents relating to the days when the forecast indicated 80% likelihood of snow. Meteorologists compared their predictions with the actual presence of snow on those days. The 80% prediction of the probability of snow can be considered very accurate when it snowed on:

a. 95%–100% of those days
b. 85%–94% of those days
c. 75%–84% of those days
d. 65%–74% of those days
e. 55%–64% of those days

GRAPHICAL FORMAT
A meteorological center wanted to determine the accuracy of its forecasts. For this, it searched documents relating to the days when the forecast showed a 70% probability of rain. Meteorologists compared their predictions with the actual presence of rain on those days. The 70% prediction of the probability of rain can be considered very accurate if it rained on:

a. 95%–100% of those days
b. 85%–94% of those days
c. 75%–84% of those days
d. 65%–74% of those days
e. 55%–64% of those days

Figure 1. *Items in verbal-numerical format and graphical format*

We registered the subjects' categorical responses to the problems (repeated measures in verbal–numerical/graphical–pictorial form) under conditions of time pressure.

We analysed the items singularly, according to other works carried out on verbal–graphical uncertainty problem-solving (e.g. Brase, 2009; Moro et al., 2011). This was expected to increase the likelihood of successfully evaluating specifically how dissimilar graphs could have a different effect on solving uncertainty and probabilistic problems.

In this investigation we applied a quasi-experimental mixed design. Indeed, each student was evaluated on both formats of the problems (formulated respectively in verbal–numerical and graphical ways). This design allows for controlling the effect of variability related to individual differences. The experimental problems were presented in a condition of time pressure; thus the subject was constrained to work within a given time limit of 90 seconds.

To analyse the obtained data we applied non-parametric statistical data analysis, specifically the McNemar Test and the Marginal Homogeneity Test.

Results

Both pairs of problems focused on tests of reasoning on the uncertainty that an event will be realized, implying the understanding of the concept of probability expressed in percentage terms.

In the first phase, we compared the replies to each pair of items in the two formats. We bring into comparison correct vs. incorrect answers given to each item in the two conditions (with and without graphics).

The obtained results are reported in the following Tables 1 and 2 under the form of contingency tables. The rows report the different types of presentation format (verbal numerical and graphical) and the columns the kinds of responses (incorrect / correct). In Table 1 we can observe the number of incorrect/correct responses to the first problem, comparing the two presentation formats. In Table 2 we reported, with the same organization of data, the observed frequencies in relation to the second pair of problems.

Table 1. *Frequencies of responses in understanding of disease risk related to surgery in the two presentation formats.*

	Types of answers		
Types of presentation format	INCORRECT	CORRECT	TOTAL
VERBAL-NUMERICAL FORMAT	41	19	60
GRAPHICAL FORMAT (EULERO-VENN)	38	129	167
TOTAL	79	148	227

Table 2. *Frequencies of responses in understanding accuracy of probabilistic weather forecasting in the two presentation formats.*

	Types of answers		
Types of presentation format	INCORRECT	CORRECT	TOTAL
VERBAL-NUMERICAL FORMAT	61	20	81
GRAPHICAL FORMAT (IDEOGRAM)	36	109	145
TOTAL	97	129	226

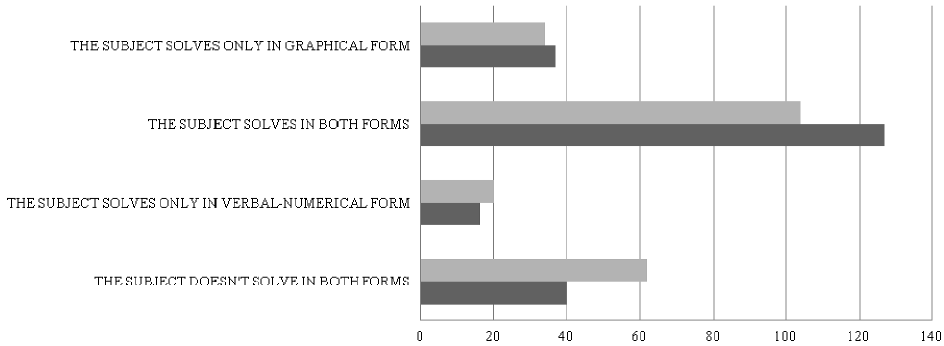
Since these two variables are repeated measures and the problems are restricted and limited only to two couples of dissimilar proofs (for each of which we wanted to compare respectively and separately the verbal-numerical and graphical form) we applied the bivariate McNemar test to each pair of problems. This allowed to detect a significance of change for both pairs of problems (respectively, p=.016 and p=.044). This test evidenced a significant difference in performance between the verbal–numerical and graphical conditions. The responses to the two pairs of items in corresponding formats are significantly different.

However, our interest was mainly focused on people who did not work well when dealing with verbal–numerical presentation of evidence, while their performance was good in the presence of a pictorial–graphical presentation. To highlight this condition, we coded the answers through new categorical variables (one for each pair of items):

1. individuals who did not give exact answers in relation to both presentation formats;
2. individuals who gave exact answers only in relation to the verbal–numerical format;
3. individuals who gave exact answers only in relation to the pictorial–graphical format;
4. individuals who gave exact answers in relation to both presentation formats.

The creation of this typological index is principally advantageous in this framework because it allows to synthesize into a single categorical variable the specificities of the two variables associated with the answer: the solution of each pair of problems in two corresponding ways. What is of particular importance is the ability to examine information in a joint and synthetic method, rather than in relation to each single variable (Marradi, 2007). By this index we can identify a single dependent categorical variable, which appropriately takes into account all potential permutations of replies to the two ways of problem presentation (verbal–numerical and graphic).

In Figure 2 we can compare the frequencies of these new categorical variables in relation to each pair of uncertainty problems.



	THE SUBJECT DOESN'T SOLVE IN BOTH FORMS	THE SUBJECT SOLVES ONLY IN VERBAL-NUMERICAL FORM	THE SUBJECT SOLVES IN BOTH FORMS	THE SUBJECT SOLVES ONLY IN GRAPHICAL FORM
understanding accuracy of probabilistic weather forecasting	62	20	104	34
understanding of disease risk related to surgery	40	16	127	37

Figure 2. *Frequencies of new categorical variables in relation to each pair of uncertainty problems*

These two distinct categorical variables, related respectively to the first and second pairs of problems, were then placed in comparison using the non-parametric Test of Marginal Homogeneity. We obtained significant differences between the two pairs of items. The number of subjects who gave correct answers only in the presence of a pictorial–graphical presentation format was higher in the first pair of items (using a pictorial–graphical presentation based on Euler-Venn diagram) compared to the second pair (using a presentation based on an ideogram) (p=.021).

Discussion

The aim of this study was to assess the role played by the type of presentation of a problem (verbal or pictorial structure) in university undergraduate students' solution of uncertainty problems. The key goal was to find some evidence regarding the occurrence of a facilitation effect of the graphical form on uncertainty problem-solving performance, as already asserted in previous research (e.g. Garcia-Retamero et al., 2011).

Our results highlighted that subjects made more accurate judgements when they used a graphical representation. Our students performed better when they based their assessment on graphs, and in particular on the Euler-Venn diagram. Indeed, on the basis of our results, it would seem that the presentation structure might have a significant effect on statistical reasoning performance. We found that graphical representation improves the performance in the resolution of uncertainty problems. This effect appears to be especially related to the use of Euler-Venn diagrams, which increases individuals' capability to individuate the correct problem solution. The Euler-Venn diagrams appear as an important instrument to promote the *transnumeration* (Wild & Pfannkuch, 1999) that has been discussed as a relevant way of improving statistical reasoning.

Our findings appear to be in line with research emphasizing that similar graphical representations can improve the resolution of some kinds of problems – in this case, uncertainty and probabilistic problems (e.g. Brase, 2009; Moro et al., 2011; Sedlmeier, 2002; Yamagishi, 2003). Furthermore, our results are interesting also in relation to an analysis of the interaction between the subjects' different skills (i.e. reasoning on uncertainty and mathematical problems).

We showed that the presence/absence of images may produce, in terms of the type of task, a specific effect on reasoning on uncertainty. These circumstances prove the usefulness of graphic representations in order to promote this reasoning and to communicate the information related to uncertainty and probability. This point could be very relevant and useful, especially when dealing with inexperienced subjects.

Undoubtedly, efficacy in the use of illustrations would be maximized if individual skills and prerequisites were carefully assessed. Assessment and contextualization of individual characteristics appear to be very useful for the improvement and enhancement of probabilistic reasoning under uncertainty. In fact, the usefulness of representations in fostering reasoning under uncertainty is affected by the structure of the task and by the individual specificities (individual–task combination; Zhu & Gigerenzer, 2006).

It is essential to say that our research has some limits. It would be useful to increase and revise the items, in order to design a greater number of items in which the probabilistic problem is presented only in a graphical format (rather than in combined verbal and graphical format). Moreover, it would be constructive to assess specific individual skills in numerical reasoning and in visuo-spatial abilities, in order to evaluate these individual characteristics' contribution to the reasoning. Finally, it would be productive to administer the problems to subjects in different age ranges.

In conclusion, we found an effect of graphical facilitation on reasoning on uncertainty that, nevertheless, depends on the graphical representation specifically employed. We suggest that probabilistic reasoning under conditions of uncertainty could be aided by highly meaningful pictorial–graphical presentations.

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Mirian Agus is a psychologist, she is working at the University of Cagliari and she is attending a Ph.D. course in methodology of the behavioral sciences at the University of Barcelona. She is interested in the study of statistical reasoning and in the application of statistical methods in the psychological field.
Contact: mirian.agus@unica.it

Maria Pietronilla Penna is a full professor of experimental psychology at the University of Cagliari and is mainly involved in the investigation of the statistical reasoning and the cognitive processes of life span in terms of behavioral studies and artificial neural networks. Contact: penna@unica.it

Ph.D. **Maribel Peró-Cebollero** is a full time Teacher in the Department of Methodology of the Behavioral Sciences in the Faculty of Psychology-University of Barcelona. She teaches subjects related with statistics and epidemiological studies in graduate and postgraduate courses. She belongs to many groups recognized by the University of Barcelona: "Grup de Recerca Tècniques Estadístiques Avançades Aplicades a la Psicologia", "Grup Innovació Docent Aprenentatge Estadística en Salut", "Institut de Recerca Cervell, Cognició i Conducta".
Contact: mpero@ub.edu

Ph.D. **Joan Guàrdia-Olmos** is Professor on Methodology of Behavioral Sciences of the Faculty of Psychology-University of Barcelona. His academic activity is related with the Statistical Data Analysis; he teaches in several grades/postgraduates courses in national and international universities. He is responsible of groups recognized by the University of Barcelona: "Institut de Recerca Cervell, Cognició i Conducta", "Grup de Recerca Tècniques Estadístiques Avançades Aplicades a la Psicologia" and "Grup Innovació Docent Aprenentatge Estadística en Salut". Contact: jguardia@ub.edu

Eliano Pessa is a full professor of general psychology at the University of Pavia. Among his many research interests there is the investigation of statistical reasoning and cognitive processes related to memory, visual perception, and decision making, both in terms of behavioural studies and computational models based on artificial neural networks. Contact: eliano.pessa@unipv.it